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Bright solitons under the influence of third-order dispersion and self-steepening effect

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Abstract

The evolution of **broad-band laser pulses** in nonlinear dispersive media, such as one-dimensional and planar waveguides, attracts a considerable attention in last decades. The well-known **nonlinear Schrodinger equation (NSE)** is one of the most commonly used in optics to describe the propagation of narrow-band light pulses, but in the frames of **ultrashort optics**, it is necessary to use the more general nonlinear amplitude equation (NAE). It works very well for nanosecond and picosecond as well as **attosecond** and **femtosecond** optical pulses. The influence of **higher orders of dispersion** and **nonlinearity** of the medium becomes significant for broad-band laser pulses. As a result it is needed to include additional terms in NAE that govern these effects.

In the present work the propagation of **bright solitons under the influence of third-order dispersion** and **self-steepening effect** in single-mode fibers is analytically and numerically studied. Such optical pulses can be observed as a result of the dynamic balance between higher orders of dispersive and nonlinear phenomena. **New exact analytical soliton solution of NAE** in the form of **cnoidal wave** is found. The solution is presented by **Jacobi elliptic delta function**. It is shown that at certain values of the parameter κ the solution can be reduced to ***sech*-soliton**.

The obtained results are important for the better understanding of the propagation of **bright optical solitons in nonlinear dispersive media** under the influence of third order of linear dispersion and self-steepening effect. They can be used in **telecommunications technology** for signal transmission across longue distances.

Key words: *nonlinear amplitude equation, optical solitons, cnoidal waves*

Introduction

The most commonly used equation in nonlinear optics happens to be the **Nonlinear Schrodinger equation** (**NSE**) [1,2]. The NSE is derived for narrow-band pulses, for which it is satisfied the condition $\boldsymbol{\Delta\omega \ll \omega_0}$. This concerns nano and picosecond laser pulses.

However, nowadays it is easy to obtain broad-band optical pulses where $\boldsymbol{\Delta\omega \approx \omega_0}$. Such light pulses are phase-modulated or they are with femto or attosecond duration. For such optical pulses, in ultrashort optics (where $T_0 \ll 1\ ps$) the influence of the **nonlinearity** and the **dispersion** are significant. It is well-known that NSE describes very well the evolution of slowly varying amplitude function of the envelope of narrow-band pulses in optical fibers but for broad-band laser pulses it is necessary to use the more general nonlinear amplitude equation NAE [3,4]. It differs from the standard NSE with two additional terms, which govern the **third order of linear dispersion (TOD)** and **dispersion of nonlinearity**.

Thus, the biggest advantage of NAE is that it can be applied in both cases – for pulses with broad-band and narrow-band spectrum.

The soliton regime of propagation of optical pulses in isotropic medium under the influence of third order of linear dispersion and dispersion of nonlinearity, described in the frames of NAE was studied by authors in [5]. New exact analytical soliton solution of NAE is found by using mathematical method described in [6].

The optical soliton is a wave packet which propagates in different nonlinear waveguide media with low losses at a constant velocity without changing its shape over significant distances.

Introduction

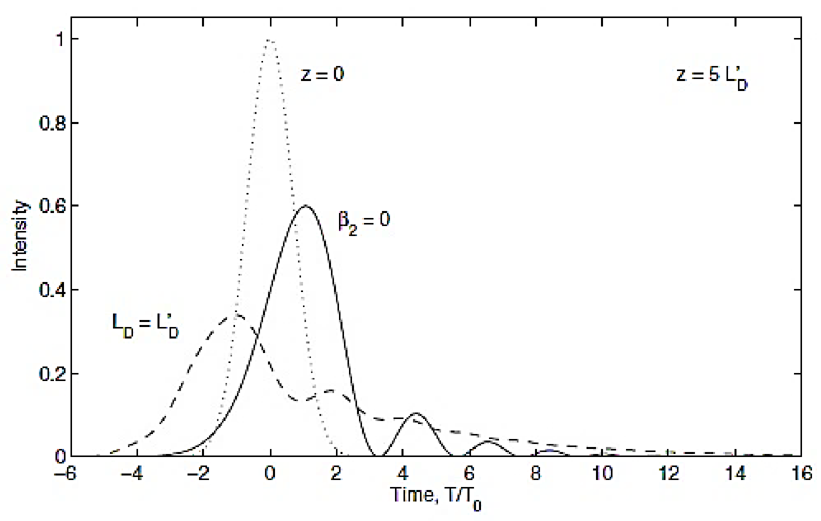

Fig.1. Gaussian pulse in the presence of TOD, $L'_D=T_0^3/|\beta_3|$ [1].

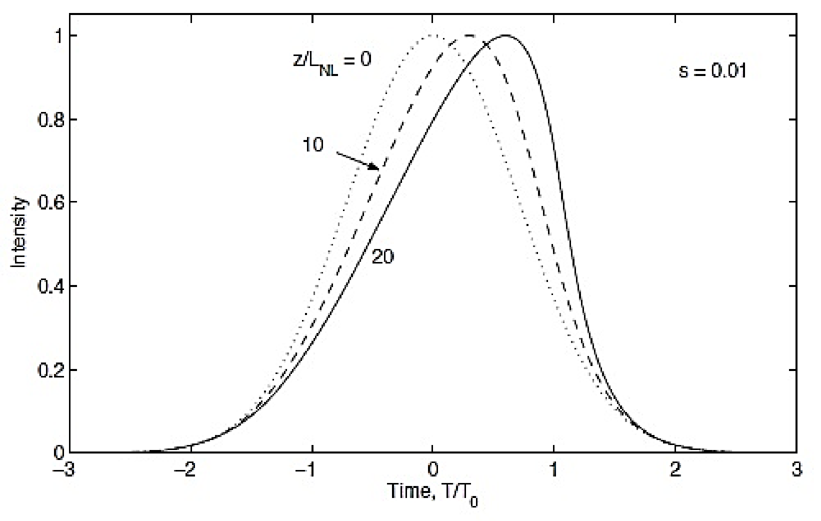

Fig.2. Dispersionless case of self-steepening of Gaussian pulse, $z=10\ L_{NL}$ and $20\ L_{NL}$, $s=1/\omega_0 T_o$ [1].

It is well known that for ultrashort optical pulses it is necessary to be included (TOD) β_3 even when group velocity dispersion $\beta_2 \neq 0$.

As a result of that the shape of the pulse becomes asymmetric with oscillatory structure on one of its edges, depending on the sign of β_3 (Fig. 1).

Self-steepening ($s=1/\omega_o T_o$) is a higher-order nonlinear effect that results from the intensity dependence of group velocity and leads to an asymmetry in the shape and spectrum of ultrashort pulses.

Their peaks shift toward the trailing edges, moving at lower speed than wings (Fig. 2).

Basic equation

We are investigating the evolution of one-dimensional optical pulses in nonlinear single mode waveguides:

$$\frac{\partial^2 A}{\partial z^2} + 2ik_0\left[\frac{\partial A}{\partial z} + \frac{1}{u}\frac{\partial A}{\partial t} + \left(n_2\frac{k_0}{\omega_0} + \frac{k_0}{2}\frac{\partial n_2}{\partial \omega}\right)\frac{\partial}{\partial t}(|A|^2 A)\right] + \frac{2i}{u}\hat{D}\frac{\partial A}{\partial t} + 2k_0\hat{D}A - \frac{1}{u^2}\frac{\partial^2 A}{\partial t^2} + n_2 k_0{}^2|A|^2 A = 0 \quad (1)$$

where A is the amplitude function of the pulse's envelope, u is pulse's group velocity, n_2 is the nonlinear refractive index, ko is the wave number, k'' and k''' are respectively the second and third order of linear dispersion and $\hat{D} = -\frac{1}{2}k''\frac{\partial^2}{\partial t^2} - \frac{ik'''}{3.2}\frac{\partial^3}{\partial t^3}$.

We are going to use "local time" coordinate system:

$$\frac{\partial^2 A}{\partial z^2} + \frac{1}{u^2}\frac{\partial^2 A}{\partial T^2} - \frac{2}{u}\frac{\partial^2 A}{\partial z\,\partial T} + 2ik_0\left[\frac{\partial A}{\partial z} - \frac{1}{u}\frac{\partial A}{\partial T} + \frac{1}{u}\frac{\partial A}{\partial T}\right] + 2ik_0{}^2 n_2\left[\frac{1}{\omega_0} + \frac{1}{2n_2}\frac{\partial n_2}{\partial \omega}\right]\frac{\partial}{\partial t}(|A|^2 A) \quad (2)$$

We are making the following substitutions: $\xi = \frac{z}{z_0}$, $\tau = \frac{T}{T_0}$, $z_0 = uT_0$, $A = |A_0|A'$

Substitutions and initial conditions

Having in mind the substitutions from the previous slide and the following conditions:

$$\frac{\alpha n_2 |A_0|^2}{2} = \gamma = \mathrm{const} \tag{3}$$

$$\left[\frac{2}{T_0\omega_0} + \frac{\omega_0}{n_2}\frac{1}{T_0\omega_0}\frac{\partial n_2}{\partial \omega}\right] = \frac{1}{T_0\omega_0}\left[2 + \frac{\omega_0}{n_2}\frac{\partial n_2}{\partial \omega}\right] = s = \mathrm{const} \tag{4}$$

$$\beta_2 = -k_0 |k''| u^2 \tag{5}$$

$$\sigma = \frac{k_0 u}{3}\frac{|k'''|}{|k''|} \tag{6}$$

Nonlinear Amplitude Equation can be presented in the form:

$$i\frac{\partial A}{\partial \xi} + \frac{1}{2\alpha}\left[\frac{\partial^2 A}{\partial \xi^2} - 2\frac{\partial^2 A}{\partial \xi\, \partial \tau}\right] + \frac{\beta_2}{2\alpha}\frac{\partial^2 A}{\partial \tau^2} + i\frac{\beta_2}{2\alpha^2}(1+\sigma)\frac{\partial^3 A}{\partial \tau^3} + is\gamma\frac{\partial}{\partial \tau}(|A|^2 A) + \gamma |A|^2 A = 0 \tag{7}$$

Mathematical method

We are going to find a solution of our basic equation (1) by following the algorithm:

1) We make the substitution *A(ξ,τ)=Φ(t)exp(iaτ+ibξ)* then the amplitude equation takes the form *(a* and *b* are constants about to be found*)*:

$$-b\Phi+\frac{1}{2\alpha}\left[-b^2\Phi-2ib\Phi'+2ab\Phi\right]+\frac{\beta_2}{2\alpha}\left[\Phi''+2ia\Phi'-a^2\Phi\right]+i\frac{\beta_2}{2\alpha^2}(1+\sigma)\left[\Phi'''+3ia\Phi''-3a^2\Phi'-ia^3\Phi\right]+is\gamma\left[3\Phi^2\Phi'+ia\Phi^3\right]+\gamma\Phi^3=0 \quad (8)$$

2) In this equation we are going to divide the real and the imaginary parts on both sides of the equality:

$$\text{Re:}\ \left|\frac{\beta_2}{2\alpha}\Phi''-\frac{3a\beta_2(1+\sigma)}{2\alpha^2}\Phi''-b\Phi-\frac{b^2}{2\alpha}\Phi+\frac{2ab}{2\alpha}\Phi+\frac{a^3\beta_2(1+\sigma)}{2\alpha}\Phi-\frac{a^2\beta_2}{2\alpha}\Phi+s\gamma a\Phi^3+\gamma\Phi^3=0\right. \quad (9)$$

$$\text{Im:}\ \left|\frac{\beta_2(1+\sigma)}{2\alpha^2}\Phi'''-\frac{2b}{2\alpha}\Phi'+\frac{2a\beta_2}{2\alpha}\Phi'-\frac{3ia^2\beta_2(1+\sigma)}{2\alpha^2}\Phi'+3s\gamma\Phi^2\Phi'=0\right. \quad (10)$$

We can reduce the order of equation (10) by integrating it. After couple of transformations and some mathematical operations we come up with the following system of equations:

$$\text{Re:}\quad \Phi''-\Phi\frac{\left(2\alpha^2b+b^2\alpha-2ab\alpha-a^3\beta_2(1+\sigma)+a^2\alpha\beta_2\right)}{\alpha\beta_2-3a\beta_2(1+\sigma)}+2\frac{\gamma\alpha^2(1-sa)}{\alpha\beta_2-3a\beta_2(1+\sigma)}\Phi^3=0 \quad (11)$$

$$\text{Im:}\quad \Phi''-\Phi\frac{\left(2\alpha b-2\alpha a\beta_2+3a^2\beta_2(1+\sigma)\right)}{\beta_2(1+\sigma)}+2\frac{\alpha^2 s\gamma}{\beta_2(1+\sigma)}\Phi^3=\frac{C}{\beta_2(1+\sigma)} \quad (12)$$

Mathematical method

3) The expressions (11) and (12) form a system of two differential equations of the same type. They refer to the same unknown function, therefore they should fully match. In order to do that, the coefficients in front of Φ, Φ'' and Φ^3 and the free term should be equal. From the equalization of (11) and (12) are made the following assumptions:

- The integration constant $C = 0$ (13)
- From the equalization of the coefficients in front of Φ^3 we find an expression for the constant a:

$$a = \frac{\alpha s - (1+\sigma)}{2s(1+\sigma)} \tag{14}$$

- From the equalization of the coefficients in front of Φ and having in mind (14) we find an expression, defining the constant b:

$$b = \left(\frac{1}{s} - \alpha\right)\left[1 \pm \sqrt{1 - \beta_2 \frac{\alpha s - (1+\sigma)}{\alpha s(\alpha s - 1)^2}}\right] \tag{15}$$

Mathematical method

4) Having in mind that the two equations in the system (11) and (12) must match, the equation for the unknown function Φ can be presented as follow:

$$\Phi'' - \Phi\frac{\left(2\alpha b - 2\alpha a\beta_2 + 3a^2\beta_2(1+\sigma)\right)}{\beta_2(1+\sigma)} + 2\frac{\alpha^2 s\gamma}{\beta_2(1+\sigma)}\Phi^3 = 0 \qquad (16)$$

This equation is of the type $y'' - (2-k^2)y + 2y^3 = 0$, where the parameter 0<<*k*<<1. When Φ(0)=1 the solution of this equation is Jacobi delta function *dn (τ,k)*.

The graphic of this function for different values of *k* is shown on Fig. 3. When *k*=1, the Jacobi delta function is transform into *sech*-function: *dn(τ,k)=sech(τ).*

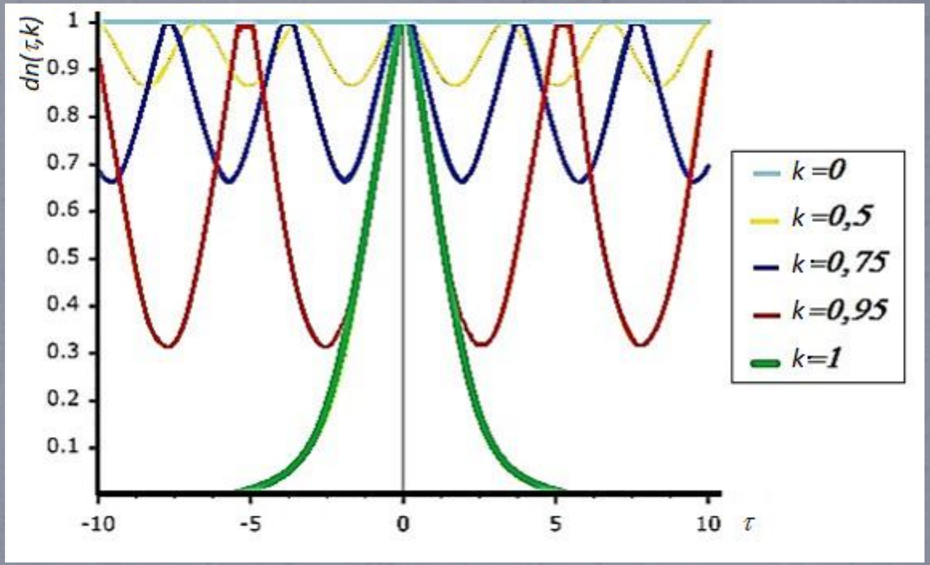

Fig. 3. Graphic of Jacobi delta function dn(τ, k) for different k.

Mathematical method

5) Considering the expressions from the previous slide and when the following conditions are applied:

$$\gamma\frac{\alpha^2 s}{\beta_2(1+\sigma)} = 1 \qquad (17)$$

$$\frac{2\alpha b - 2\alpha a\beta_2 + 3a^2\beta_2(1+\sigma)}{\beta_2(1+\sigma)} = 2 - k^2 \qquad (18)$$

the solution of the equation (16) can be presented by Jacobi's elliptical delta function:

$$\Phi(\tau)=dn(\tau,k) \qquad (19)$$

6) Going back trough all the substitutions and assumptions, we find the following solution of the basic equation (7):

$$A(\xi,\tau)=dn(\tau,k)exp(ia\tau+ib\xi), \qquad (20)$$

where the cosntants a and b are defined by the expressions (14) and (15).

Mathematical method

It is clearly seen from the expression (18) that the value of the constant *k* is defined by the parameters of the media and the optical pulse. When *k*=1 a **soliton solution of equation (16)** is obtained:

$$\Phi(\tau)=sech(\tau) \tag{21}$$

From the expression (17) we determine the initial intensity which is needed to form a soliton:

$$|A_0|^2 = \frac{|k''|}{T_0^2}\frac{(1+\sigma)}{k_0 n_2} = |A_0|^2_{shr}(1+\sigma) \qquad |A_0|^2 > |A_0|^2_{shr} \tag{22}$$

Here $|A_o|^2{}_{shr}$ is the initial intensity for forming a soliton, based on the NSE.

The condition (18) gives the relation between the parameters of the medium and the parameters of the initial impulse, for which it can be formed a soliton (when *k*=1):

$$2\alpha b - 2\alpha a\beta_2 + 3a^2\beta_2(1+\sigma) = \beta_2(1+\sigma) \tag{23}$$

Approximate solution

When we assume that $\partial n_2 / \partial \omega \approx 0$.In this case the higher order of nonlinearity is weak. Therefore, the self-steepening parameter is equal to s=2/α , where α is the number of oscillations under the envelope of the optical pulse.
Having in mind that the expressions for the constants a and b will take the following approximate form:

$$a \approx \frac{\alpha(1-\sigma)}{4(1+\sigma)} \tag{24}$$

$$b \approx -\frac{\alpha}{2}\left[1 \pm \sqrt{1-\beta_2(1-\sigma)}\right] \tag{25}$$

In our investigations T_0 is the initial duration of laser pulses and ω_0 is the main frequency.

Numerical calculations

For laser pulse with carrier wavelength $\lambda=1{,}45\ \mu m$ propagating in silica fiber with the following parameters: $n_0=1{,}453$, $n_{gr}=1{,}467$, $|k''|=1{,}68.10^{-26}s^2/m$, $|k'''|=1{,}17.10^{-40}s^3/m$, it is defined that $k_0\approx 4{,}33.10^6\ m^{-1}$. In order to be observed bright soliton (k=1) it is necessary the initial duration of the pulse to be $T_0\approx 7{,}5\ fs$ and $\alpha\approx 4$. It turns out that the initial intensity needed for formation of fundamental soliton is approximately three times bigger of that of the typical Schrodinger soliton.
The graphic of the soliton solution (20) is presented of Fig. 4.

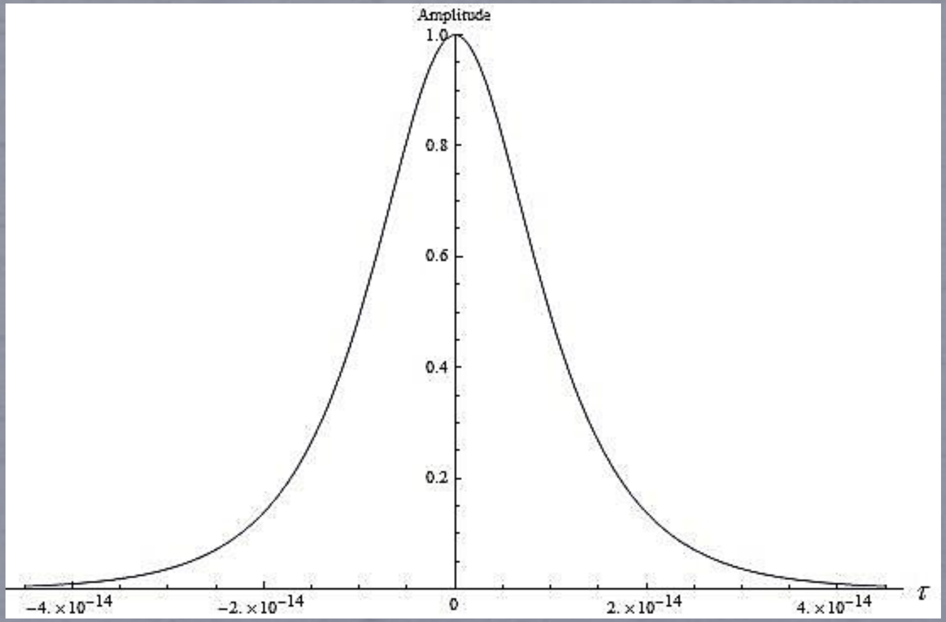

Fig.4. Bright optical soliton, given by the solution (20).

Conclusions

❖ In the present work it is investigated the evolution of **broad-band laser pulses** in nonlinear dispersive media.

❖ The propagation of **bright optical solitons** under the influence of **third-order of linear dispersion** and **self-steepening effect** in single-mode fibers is analytically and numerically studied.

❖ **New exact analytical soliton solution of NAE** in the form of **cnoidal wave** is found. The solution is presented by **Jacobi elliptic delta function**.

❖ It is shown that at certain values of the parameter k the solution can be reduced to ***sech*-soliton**.

❖ There have been made **graphics** of the obtained solutions.

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Thank you for the attention!